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On effective spacetime dimension in the Hořava–Lifshitz gravity

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ABSTRACT

In this manuscript we explicitly compute the effective dimension of spacetime in some backgrounds of Hořava–Lifshitz (H–L) gravity. For all the cases considered, the results are compatible with a dimensional reduction of the spacetime to $d + 1 = 2$, at high energies (ultraviolet limit), which is confirmed by other quantum gravity approaches, as well as to $d + 1 = 4$, at low energies (infrared limit). This is obtained by computing the free energy of massless scalar and gauge fields. We find that the only effect of the background is to change the proportionality constant between the internal energy and temperature. Firstly, we consider both the non-perturbative and perturbative models involving the matter action, without gravitational sources but with manifest time and space symmetry breaking, in order to calculate modifications in the Stephan–Boltzmann law. When gravity is taken into account, we assume a scenario in which there is a spherical source with mass M and radius R in thermal equilibrium with radiation, and consider the static and spherically symmetric solution of the H–L theory found by Kehagias–Sfetsos (K–S), in the weak and strong field approximations. As byproducts, for the weak field regime, we used the current uncertainty of the solar radiance measurements to establish a constraint on the ω free parameter of the K–S solution. We also calculate the corrections, due to gravity, to the recently predicted attractive force that black bodies exert on nearby neutral atoms and molecules.

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1. Introduction

The Hořava–Lifshitz (H–L) gravity is an alternative proposal to quantum gravity theory, recently presented in the literature, which is power-counting renormalizable [1–3]. The cost to ensure this renormalizability is to ignore the local Lorentz symmetry incorporated in Einstein's theory of gravitation and to consider different kinds of spatial and temporal scaling at very short distances, *i.e.*, in the ultraviolet (UV) regime, with this symmetry accidentally emerging in the opposite situation, namely, at the infrared (IR) regime [4]. According to this novel approach, at the UV regime an anisotropic scaling occurs, in such a way that time transforms as $t \rightarrow b^z t$ and space as $x^i \rightarrow b x^i$, where z is a dynamical critical exponent that goes to unity at large distances, which means that the validity of General Relativity is restored, at the IR scale.

The possible values of the critical exponent z establish a connection between the H–L theory and the number of dimensions that the Universe must possess at all scales. In fact, Hořava [3] calculates the dimension seen by a diffusion process, the spectral dimension d_s , as being equal to 2 at the UV scale, which is compatible with other approaches of quantum gravity [5–7]. At the opposite scale, which means in the IR regime, the spectral dimension is $d_s = 4$ [3]. The spectral dimension as a continuous function of time which links these two amounts, in the context of the H–L theory, was obtained in [8]. A general discussion about the dependence of the dimensionality with the scale (“evolving” or “vanishing” dimensions), in the context of several theories, can be found in [9]. Hořava still presents [3] heuristic scaling arguments based on the free energy of a system consisting of free massless fields to show that the effective number of topological dimensions is again equal to 2, since that the free energy found is proportional to the square of temperature.

Studies concerning H–L gravity were accomplished in several contexts, as in the analysis of the Casimir effect [11,12] and in the study of spacetime stability [13], as well as in some astrophysical

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and cosmological models [14–16], including those ones which predict a multiverse [17]. It was shown that H–L gravity is weaker than General Relativity even at high energies, generating a bouncing cosmology and an accelerating universe, naturally incorporating dark energy [18,19]. From the point of view of Astrophysics, some static and spherically symmetric black hole-type solutions of H–L gravity were found, and the simplest one was proposed by Kehagias and Sfetsos (K–S) [20], whose IR limit gives us the Schwarzschild solution. Other exact spherically symmetric solutions for the H–L gravity can be found in [21–24].

The connection between the phenomenology involving gravity and thermodynamics was studied in the early seventies of past century [26,27], and reexamined more recently [28,29]. In this context, black holes seem to be the objects of the universe in which these connections become stronger, and any modifications in the laws derived from thermodynamics due to gravitational effects must necessarily consider these structures. In this scenario, black body radiation is emblematic in view of the possible role played by that phenomenon in the construction of a quantum theory of gravity.

In this paper, we calculate explicitly the number of effective topological dimensions of the Universe by analyzing modifications in the laws of the black body radiation due to the H–L theory, at both UV and IR scales with and without gravitation sources. It is supposed that the thermal radiation comes from the dynamics of a free massless scalar field. We then calculate the Helmholtz free-energy associated with the field for the cases under consideration. Firstly, the non-perturbative model in which, due to the spacetime anisotropy required by H–L theory, the spatial derivatives of the scalar field are implemented in the action with higher orders than the temporal one.

In what follows, we will take into account the perturbative model where, besides these derivatives, we add one more term which depends on the first order spatial derivative. We use the models presented in [11] to obtain the dependence of the Helmholtz free-energy with the temperature and to determine the effective number of spacetime dimensions. Next we consider a spherical gravitational source, by considering the weak and strong field approximations of the K–S solution, in both IR and UV regimes. Finally, for the weak field regime, we will obtain constraints on the K–S ω parameter from the uncertainty in the measurements of the solar radiance. In view of the both General Relativity and H–L theories we will also calculate corrections to the thermal shift in the energy of the fundamental state of the hydrogen atom, known as dynamical Stark effect, obtaining the anomalous force of attraction upon it due to the black body radiation.

The paper is organized as follows. In Section 2, we determine modifications in the laws of the black body radiation in the context of the H–L theory without considering the presence of a gravitational field. In Section 3, we do similar calculations to the previous section, but now taking into account the presence of a gravitational field. In Section 4 we obtain the force of attraction corrected by effects due to classical and H–L gravity. Finally, in Section 5 we present the concluding remarks.

2. Black body radiation without gravitational sources

2.1. Non-perturbative model

In the H–L theory, the violation of the Lorentz symmetry at high energies comes from the different forms of scaling time and space. This feature generates changes in the action of a free massless scalar field with respect to that one described in Minkowski

spacetime. In this sense, the more general action for such a field can be written as

$$S = \int dt d^d x \frac{1}{2} \left[(\partial_t \phi)^2 - \sum_{s=1}^z g_s (\partial^{i_1} \partial^{i_2} \dots \partial^{i_s}) \times \phi (\partial_{i_1} \partial_{i_2} \dots \partial_{i_s}) \phi \right]. \quad (1)$$

The non-perturbative model is that one in which the Eq. (1) only contains the terms where $s = z$ and $g_z = \ell^{2(z-1)}$, where ℓ is a characteristic length. Thus the field equation, according to this model, is given by

$$\partial_t^2 \phi - \ell^{2(z-1)} \nabla^{2z} \phi = 0. \quad (2)$$

Considering an oscillatory field of the form $\phi(\vec{x}, t) = C \exp(i\vec{k} \cdot \vec{x} - i\omega t)$, we get the eigenfrequencies given by $\omega = \ell^{(z-1)} k^z$, where $k = \sqrt{k_x^2 + k_y^2 + k_z^2}$.

The black body radiation is described by the Stephan–Boltzmann law, which can be found from the Helmholtz free-energy density, expressed by [31]

$$f_{bb}(T) = 2k_B T \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \log\{(1 - e^{-\tilde{\beta} k^z})\}, \quad (3)$$

where $\tilde{\beta} = \ell^{z-1}/k_B T$. The factor 2 was introduced to account for the two modes of polarization of the electromagnetic waves. When $z = 1$, namely, in the low-energies domain, this free-energy density is

$$f_{bb}(T) = -\frac{\pi^2 (k_B T)^4}{45}, \quad (4)$$

which leads to the law of Stephan–Boltzmann, from which we obtain the energy density of the black body, given by

$$u_{bb}(T) = -T^2 \frac{\partial (f_{bb}/T)}{\partial T} = \frac{\pi^2 (k_B T)^4}{15}, \quad (5)$$

as it should be in the IR limit and in absence of a gravitational field.

On the other hand, if we put $z = 3$ in Eq. (3), which corresponds to high-energies scale, the free-energy density will be now given by

$$f_{bb}(T) = -u_{bb}(T) = -\frac{(k_B T)^2}{18\ell^2}. \quad (6)$$

It is known that the Stephan–Boltzmann law in a d -dimensional flat space yields a black body energy density proportional to T^{d+1} [32,33]. Thus, by the above expression, we can see that the effective dimension of the spacetime at high-energies scales is $d + 1 = 2$, according to the Hořava–Lifshitz theory. This fact will not change when we take into account the gravitational field. Such specific dimensional reduction at UV scale is compatible with other approaches of quantum gravity [5,6]. In fact, this dependence with the square of temperature works for any d -dimensional flat space, provided that $z = d$ in order to have the renormalizability of the theory. In this case, the measure of the integral (3) is $[\pi^{-d/2} 2^{-d} \Gamma(d/2 + 1)] k^{d-1}$, where $\Gamma(x)$ is the Gamma function and the black body energy density is exactly given by

$$u_{bb}(T) = \frac{2^{-d} \pi^{(4-d)/2} \ell^{1-d}}{3\Gamma(d/2 + 1)} (k_B T)^2. \quad (7)$$

For an arbitrary temperature, a numerical analysis of the above equation reveals that the energy density has a minimum for $d = 3$ when $\ell \approx 0.2\ell_P$, with ℓ_P being the Planck length, and thus extending the UV domain of the H–L theory to trans-Planckian scales [10].

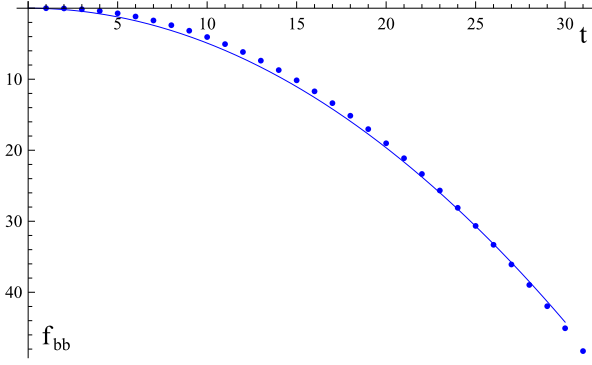


Fig. 1. Helmholtz free-energy density of the black body radiation as a function of $t = k_B T$, in the perturbative model of the H-L theory.

2.2. Perturbative model

Regarding the perturbative model, which consists of adding to the action of the non-perturbative model a term proportional to the spatial derivatives of first order, in such a way that at IR scales the higher order derivatives are neglected, the dispersion relation is given by $\varpi = k\sqrt{1 + (\ell k)^{2(z-1)}}$ [11]. In this case, the Helmholtz free-energy of the black body radiation will be, therefore,

$$f_{bb}(T) = 2k_B T \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \log \left\{ 1 - \exp \left[-\beta k \sqrt{1 + (\ell k)^{2(z-1)}} \right] \right\}, \quad (8)$$

where $\beta = (k_B T)^{-1}$. It becomes evident that, in the IR regime, namely when $z = 1$, the free-energy has the same temperature dependence as the one obtained in 4-dimensional Minkowski space-time.

At UV scale, specifically for $z = 3$, the integration of (8) is not analytically soluble, and thus we have depicted, in Fig. 1, some values of f_{bb} as a function of $t = k_B T$, for $\ell = 1$, obtained numerically. In the same graph, we fit the curve that best represents this function, which was found to be $f_{bb} \simeq -0.049117t^2$.

Then, due to the quadratic dependence of the black body density energy with the temperature, the same conclusions valid for the non-perturbative model are applied here, by considering $z = 3$.

3. Black body radiation due to a spherical gravitational source

The K-S black hole solution is given by the metric [20]

$$ds^2 = f_{KS}(\rho) dt^2 - \frac{d\rho^2}{f_{KS}(\rho)} - \rho^2 d\Omega^2, \quad (9)$$

where

$$f_{KS}(\rho) = 1 + \omega \rho^2 \left(1 - \sqrt{1 + \frac{4M}{\omega \rho^3}} \right), \quad (10)$$

with ω being a free parameter of the H-L theory and ρ a radial coordinate. It is easy to note from (9) and (10) that, in the coordinates system used, the K-S solution has two event horizons, one at $\rho_{out} = M(1 + \sqrt{1 - 1/2\omega M^2})$ and another at $\rho_{int} = M(1 - \sqrt{1 - 1/2\omega M^2})$.

The K-S solution represents a black hole if $\omega M^2 \geq 1/2$, while for $0 < \omega M^2 < 1/2$ it is a singularity without event horizons (naked singularity). In the limit $\omega \rightarrow \infty$, where ω is the H-L parameter, it reduces to the Schwarzschild solution.

3.1. Weak-field approximations

When $\omega \rightarrow \infty$ (IR limit), the metric coefficients of Eq. (9) tend to those ones of the Schwarzschild solution, i.e., $f_{KS}(\rho) \rightarrow f_{Sch}(\rho) = 1 - 2M/\rho$, which corresponds to the vacuum solution generated by a static spherical source. Thus, this result is consistent with the validity of the General Relativity at IR scale. Here we will take into account the static background gravitational field described by the K-S solution, whose metric contains an irremovable singularity at $\rho = 0$. We suppose that the radius R of the spherical body is much larger than the radii of the event horizons. In other words, we will work in a weak field approximation.

We already have emphasized that our approach will occur close to the IR limit ($z \approx 1$, $\omega \gg 1$). Thus, we will make an expansion in the metric coefficient given by Eq. (10) so that

$$f_{KS}(\rho) \approx 1 - \frac{2M}{\rho} + \frac{2M^2}{\omega \rho^4}. \quad (11)$$

The last term in (11) is what effectively incorporates the properties of Hořava–Lifshitz gravity. We can note at a glance that the attractive post-Newtonian potential is corrected by the repulsive term $M^2/\omega \rho^4$.

For our proposes, it is convenient the use of isotropic coordinates. Thus, it is necessary to find a transformation in the radial coordinate, $\rho = \rho(r)$, so that the K-S solution has the form $ds^2 = g(r)dt^2 - h(r)(dr^2 + r^2 d\Omega^2)$. Following [34], a suitable form of the metric is given by

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{2M^2}{\omega r^4} \right) dt^2 - \left(1 - \frac{2M}{r} + \frac{M^2}{2\omega r^4} \right)^{-1} \times (dr^2 + r^2 d\Omega^2), \quad (12)$$

where we have introduced the isotropic coordinates in order to use the rectangular coordinates [12].

Now, let us calculate the eigenfrequencies of the massless scalar field at the surface $r = R$ of the spherical gravitational source, in both the weak field approximation and IR regime of the K-S solution. To do this, we will rewrite the metric (12) in the following form

$$ds^2 = (1 + 2b\Phi'_0)dt^2 - (1 - 2\Phi'_0)(dx^2 + dy^2 + dz^2), \quad (13)$$

where $b = 1 - \frac{3M}{4\omega R^3}$ and $\Phi'_0 = -\frac{M}{R} + \frac{M^2}{4\omega R^4}$.

The general covariant Klein–Gordon equation for the massless scalar field in the spacetime given by equation (13) is written as

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu) \Psi = [1 - 2(b+1)\Phi'_0] \partial_t^2 \Psi - \ell^{2(z-1)} \nabla^2 \Psi = 0. \quad (14)$$

Considering plane wave solutions we can find that the eigenfrequencies ϖ are given by the expression $\varpi = \ell^{z-1} \left(1 - \frac{2M}{R} + \frac{5M^2}{4\omega R^4} \right) k^z$.

The Helmholtz free-energy density of the black body radiation is

$$f_{bb}(T) = 2k_B T \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \log \left[1 - e^{-\tilde{\beta} k^z} \right], \quad (15)$$

where the factor 2 once more was introduced to account for the two polarization modes of the electromagnetic field and $\tilde{\beta} = \ell^{z-1} \frac{[1 + \Phi'_0(b+1)]}{k_B T}$. Comparing Eq. (15) with our previous result obtained in Eq. (3), we get for $z = 1$ [12]

$$u_{bb}(T) = \frac{\left(1 + \frac{6M}{R} - \frac{15M^2}{4\omega R^4}\right) \pi^2 (k_B T)^4}{15}. \quad (16)$$

On the other hand, if we put $z = 3$ in Eq. (15), which corresponds to high-energies scale, we get

$$u_{bb}(T) = \left(1 + \frac{6M}{R} - \frac{15M^2}{4\omega R^4}\right) \frac{(k_B T)^2}{18\ell^2}. \quad (17)$$

Notice that, turning off the gravity, equations (16) and (17) recover our previous results obtained in Section 2. Therefore for the weak field approximation we again obtain that, for $z = 1$, the spacetime effective dimension equals 4. For $z = 3$ we get the expected dimensional reduction to $d = 1$. From Eq. (16), we can also define an effective temperature given by

$$T_{eff} = \left(1 + \frac{3M}{2R} - \frac{15M^2}{16\omega R^4}\right) T. \quad (18)$$

If we consider just up to M/R corrections (due to Einstein's gravity in its post-Newtonian limit), we can see that the intensity of the black body radiation is slightly augmented. With respect to the Sun, for example, this represents an increment of 0.0012 per cent comparing to a black body at the same temperature in Minkowski space. On the other hand, the correction term associated to the H-L gravity represents only a very low attenuation of that radiation. Based on the current uncertainties in measurements of the solar radiance (≈ 0.02 [36]), we can estimate a lower limit for the K-S parameter as being $\omega \geq 3 \times 10^{-31} \text{ cm}^{-2}$.

3.2. Strong-field approximations

The K-S metric at the high-energies scale, in which $\omega \ll 1$, is given by [21]

$$ds^2 = \left[1 - 2(\omega M \rho)^{1/2}\right] dt^2 - \left[1 - 2(\omega M \rho)^{1/2}\right]^{-1} d\rho^2 - \rho^2 d\Omega^2. \quad (19)$$

Taking into account an isotropic reference frame such that $r = r(\rho)$ we can write Eq. (19) as

$$ds^2 = g(r) dt^2 - f(r)^2 (dr^2 + r^2 d\Omega^2), \quad (20)$$

where $f(r) = \rho/r$ and

$$f(r) dr = \frac{d\rho}{\sqrt{1 - 2(\omega M \rho)^{1/2}}}. \quad (21)$$

Integrating this and choosing an appropriate constant of integration, we have

$$r(\rho) = M \left(\frac{\sqrt{1 - 2(\omega M \rho)^{1/2}} - 1}{\sqrt{1 - 2(\omega M \rho)^{1/2}} + 1} \right)^2, \quad (22)$$

and

$$\rho(r) = \frac{r^2}{\omega M (M - r)^2}, \quad (23)$$

such that $g(r) = 1 - 2r/(M - r)$ and $f(r) = r/\omega M (M - r)^2$.

The metric (20) can be written, up to $\mathcal{O}(r/M)$, as

$$ds^2 \approx \left(1 - \frac{2r}{M}\right) dt^2 - \frac{r^2}{\omega^2 M^6} \left(1 + \frac{4r}{M}\right) (dr^2 + r^2 d\Omega^2). \quad (24)$$

This metric must be equivalent to that one of Eq. (19) in the limit of $\omega \rightarrow 0$, provided that the limits $r \rightarrow 0$ and $(r/\omega M^3) \rightarrow 1$ are satisfied.

Let us now write the massless scalar covariant Klein–Gordon equation in the spacetime given by Eq. (24), taking $r = R$. Thus, we get

$$\left(1 + \frac{6R}{M}\right) \frac{R^2}{\omega^2 M^6} \partial_t^2 \Psi - \ell^{2(z-1)} \nabla^{2z} \Psi = 0, \quad (25)$$

where ℓ is the characteristic length [11].

Writing the dispersion relation for the massless scalar field using the ansatz $\Psi = C \exp(i\mathbf{k} \cdot \mathbf{x} - i\omega t)$, we obtain $\omega = \frac{\ell^{z-1} \omega M^3}{R} \left(1 + \frac{6R}{M}\right)^{-\frac{1}{2}} k^z$. In this case, the Helmholtz free-energy density is given by

$$f_{bb}(T) = 2k_B T \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \log \left[1 - e^{-\tilde{\beta} k^z}\right], \quad (26)$$

where $\tilde{\beta} = \ell^{z-1} \omega M^3 (1 + 6R/M)^{-\frac{1}{2}} / (k_B T R)$. Following the same steps as in the previous sections we get for $z = 3$

$$u_{bb}(T) = \frac{R}{18\ell^2 \omega M^3} \left(1 + \frac{6R}{M}\right)^{\frac{1}{2}} (k_B T)^2, \quad (27)$$

and for $z = 1$

$$u_{bb}(T) = \frac{R}{\omega M^3} \left(1 + \frac{6R}{M}\right)^{\frac{1}{2}} \frac{\pi^2 (k_B T)^4}{15}, \quad (28)$$

which are the expected results for the UV and IR regimes.

4. The “black body force”

The universal properties of the black body radiation depend on its temperature. Such radiation has always been thought to have a repulsive effect on other bodies situated near it, via momentum transference of the emitted photons. Studying the shift in the spectral lines of the hydrogen atom placed in a thermal bath, it was shown that the black body radiation exerts another quite novel and counter-intuitive attractive force on nearby neutral atoms and molecules, which is even more intense than the repulsive radiation pressure or the gravity generated by the black body. This interaction was described by the first time in [30] and according to those authors the attractive potential associated to the force exerted on a hydrogen atom in its fundamental state, placed at a distance r from the center of a spherical black body of radius R that emits radiation at a temperature T is, in natural units,

$$V_{bb}(r) = -\frac{3\pi^2 (k_B T)^4}{10(e m_e)^3} \left(1 - \frac{\sqrt{r^2 - R^2}}{r}\right), \quad (29)$$

where e and m_e are the electric charge and the mass of the electron, respectively. The last factor in the parenthesis is the solid angle through which the atom sees the source.

It is worth point out that a modification in the solid angle due to the spacetime curvature takes place here. The solid angle that appears in the equation above depends on the radial distances, which are defined now according to the g_{rr} coefficients that appear in both interior and exterior solutions, with the latter given by Eq. (12). In the case of the General Relativity, the simplest spherically symmetric interior solution is given by the Oppenheimer–Snyder metric [35], in which $g_{rr} = \left(1 - \frac{r^2}{R_0^2}\right)^{-1}$,

where $R_0 = 3c^2/8\pi G \rho_0$, with ρ_0 being the density of the ideal fluid that constitutes the body. For the Sun, whose average density is $\rho_0 \sim 10^3 \text{ kg/m}^3$, $R_0 \sim 10^{26} \text{ m}$, we have that $r \leq R$ implies $r \ll R_0$, and therefore $g_{rr}^{\text{in}} \simeq 1$. We will assume the same in the

H–L theory since that in this approach the gravitational interaction is weaker than that one of Einstein's theory.

In the weak-field approximation that we are using here, the solid angle seen by the atom will be modified by means of the radial distance from the source center up to the atom, which is expressed as

$$\begin{aligned} r \rightarrow \bar{r} &= \int_0^R \sqrt{g_{rr}^{in}} dr + \int_R^r \sqrt{g_{rr}^{ex}} dr \\ &= r + M \ln\left(\frac{r}{R}\right) - \frac{M^2}{12\omega} \left(\frac{1}{R^3} - \frac{1}{r^3}\right). \end{aligned} \quad (30)$$

The source radius R remains unaltered since that $g_{rr}^{in} \simeq 1$, as we have already seen. The temperature that we will consider here is the effective one given by Eq. (18). This will allow us to include corrections in the effect due to both the classical and quantum gravity theories, which are under consideration in this paper. We have therefore

$$V_{bb}(r) = -\frac{3\pi^2 \left(1 + \frac{6M}{R} - \frac{15M^2}{4\omega R^4}\right) (k_B T)^4}{10(em_e)^3} \left[1 - \frac{\sqrt{\bar{r}^2 - R^2}}{\bar{r}}\right], \quad (31)$$

where \bar{r} is given by Eq. (30). Thus, the attractive force is intensified by the Einstein's gravity (≈ 0.0012 per cent) and attenuated by the Hořava–Lifshitz gravity, independently of the atom distance from the black body center.

5. Concluding remarks

In this paper we have analyzed implications on the black body radiation which is derived from the dynamics of massless scalar and gauge fields in two different situations, namely, with and without a gravitational source in the framework of the Hořava–Lifshitz theory. In the gravity free case, the study was based on both the non-perturbative and perturbative models for the field action, following a recent published result [11]. For the case with source we have supposed the black body having mass M and radius R generating a static, spherically symmetric and asymptotically flat background gravity described by the Kehagias–Sfetsos solution. For the latter analysis, it was considered the weak and strong field approximations.

We found the Helmholtz free-energy of the black body in order to determine its gravitationally corrected energy density. In all cases considered we find that, at IR scale, when the space and time symmetry breaking parameter is $z = 1$, the black body energy density is proportional to T^4 , which corresponds to a Universe with spacetime dimensionality equal to 4. In UV regimen, when $z = 3$, we find that the black body energy density is proportional to T^2 , compatible with an effective number of dimensions equal to 2, which is obtained through other quantum gravity theories. It is worth notice that, specifically for the sourceless case, we showed that the quadratic dependence of the free-energy density with the temperature is independent of the topological dimension d , when we impose $z = d$ to warrant the power-counting renormalizability of the theory at UV scale. With this analysis, we have made more rigorous the argument based on anisotropic scaling of the free energy used by Hořava in [3].

Applying the above results to the Sun, we verified that the Einstein's gravity intensifies the solar radiance of about 0.0012

per cent as compared with the value obtained in flat spacetime. The term associated to the Hořava–Lifshitz gravity provides a lower bound for the free parameter $\omega \geq 10^{-31} \text{ cm}^{-2}$, based on the current uncertainties in the measurements of the solar radiance (≈ 0.02 [36]). If we consider that the Sun is not a perfect black body, then a substantial part of this measurement error comes from that assumption, what can contribute to improve the constraint upon ω , approximating it of that one obtained via classical tests of General Relativity [25], in which $\omega \geq 10^{-27} \text{ cm}^{-2}$.

Finally, regarding the weak-field and IR regime, we found the corrections due to gravity to the attractive force on neutral atoms that are under the action of the black body radiation, according to a recently published work [30]. When the atom is very close to the black body spherical surface, Eq. (29) shows that the force on the particle tends to infinity, as in the flat spacetime case. In the general situation, our correction indicates that this force will vary with the source radius, which is considerably different from that case. And if we consider the above mentioned lower bound $\omega \geq 10^{-27} \text{ cm}^{-2}$, the attenuation of the black body force caused by the Hořava–Lifshitz gravity cannot be greater than 0.00007 per cent.

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